Extra Credit 4

You have 25 horses, and you want to pick the fastest 3 horses out of those 25. In each race, only 5 horses can run at the same time because there are only 5 tracks. What is the minimum number of races required to find the 3 fastest horses without using a stopwatch?

Short Answer: 7 Races

Long Answer: 5 horses can race at a time. Then 5 number of races need to be conducted to race all 25 horses once in a batch of 5 horses each time.

Race 1: The fastest horse of first race is A1 and slowest A5.

Race 2: The fastest horse of second race is B1 and slowest is B5.

Race 3, 4 and 5: Similarly, horses for the third, fourth and fifth race are C1 to C5, D1 to D5, E1 to E5 respectively

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1st Race | A1 | A2 | A3 | A4 | A5 |
| 2nd Race | B1 | B2 | B3 | B4 | B5 |
| 3rd Race | C1 | C2 | C3 | C4 | C5 |
| 4th Race | D1 | D2 | D3 | D4 | D5 |
| 5th Race | E1 | E2 | E3 | E4 | E5 |

Till now 5 races are over.

To find out the fastest among of all 25 horses, a race needs to conduct between fastest horse of each 5 race that is between A1, B1, C1, D1, E1.

**6th Race – A1, B1, C1, D1, E1**

**Winner of 6th Race is A1 which takes First Position.**

Let's say A1 is the fastest then that horse is also fastest among all 25.

B1, C1, D1 and E1 are ranked as 2nd, 3rd, 4th, and 5th in this 6th race.

Now we have found out the fastest horse from the result of 6th race and now we need to find out the second and third ranked horse of all 25.

The 4th and 5th ranked horses of each first 5 races are eliminated for the position of 2nd and 3rd because they are slower than the respective 1st 2nd and 3rd winners.

**After 5 Races (yellow are eliminated)**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1st Race | A1 | A2 | A3 | A4 | A5 |
| 2nd Race | B1 | B2 | B3 | B4 | B5 |
| 3rd Race | C1 | C2 | C3 | C4 | C5 |
| 4th Race | D1 | D2 | D3 | D4 | D5 |
| 5th Race | E1 | E2 | E3 | E4 | E5 |

The 4th ranked horse D1, and 5th ranked E1 are also eliminated because they are slower than B1 and C1 in the 6th race.

The D2, D3 are slower than D1 are also eliminated.

The E2, E3 are slower than E1 are also eliminated.

**After the 6th Race (yellow are eliminated)**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1st Race | A1 | A2 | A3 | A4 | A5 |
| 2nd Race | B1 | B2 | B3 | B4 | B5 |
| 3rd Race | C1 | C2 | C3 | C4 | C5 |
| 4th Race | D1 | D2 | D3 | D4 | D5 |
| 5th Race | E1 | E2 | E3 | E4 | E5 |

**7th Race – A2, A3, B1, B2, C1**

The sequence of 1st,2nd and 3rd can now either of these three cases

Case 1: A1, A2, A3

Case 2: A1, B1, B2

Case 3: A1, B1, C1

Now we will eliminate B3 because slower than both B1 and B2. Also eliminate C2 and C3 as they both are slower than C1.

A final 7th race will be between A2, A3, B1, B2 and C1.

The winner and runners up of this 7th race will take the 2nd and 3rd position among all 25 horses.

The answer is 7 races.

To summarize all steps.

Step 1: 5 races of all 25 horses (5 at a time)

Step 2: 6th race will be between 5 fastest horses of first 5 races to find out the first position horse.

Step 3: Elimination of many horses who can't be in the contest for the eligibility of 2nd and 3rd position.

Step 4: 7th race will be to find out the 2nd and 3rd position.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1st Race | A1 | A2 | A3 | A4 | A5 |
| 2nd Race | B1 | B2 | B3 | B4 | B5 |
| 3rd Race | C1 | C2 | C3 | C4 | C5 |
| 4th Race | D1 | D2 | D3 | D4 | D5 |
| 5th Race | E1 | E2 | E3 | E4 | E5 |
| 6th Race | A1 | B1 | C1 | D1 | E1 |
| 7th Race | A2 | A3 | B1 | B2 | C1 |

**Assuming A1 is ranked one in the 6th race**